

## 5.8 Solving Radical Inequalities

**Radical Inequality:** Inequality that has a variable in the radicand.

To Solve Radical Inequalities:

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.

Step 2 Solve the inequality algebraically.

Step 3 Test values to check your solution.

Why?

**Example**

Solve  $5 - \sqrt{20x + 4} \geq -3$

Since the radicand of a square root must be greater than or equal to zero, first solve

$$20x + 4 \geq 0.$$

$$20x + 4 \geq 0$$

$$20x \geq -4$$

$$x \geq -\frac{1}{5}$$

Now solve  $5 - \sqrt{20x + 4} \geq -3$ .

$$5 - \sqrt{20x + 4} \geq -3 \quad \text{Original inequality}$$

$$\sqrt{20x + 4} \leq 8 \quad \text{Isolate the radical.}$$

$$20x + 4 \leq 64 \quad \text{Eliminate the radical by squaring each side.}$$

$$20x \leq 60 \quad \text{Subtract 4 from each side.}$$

$$x \leq 3 \quad \text{Divide each side by 20.}$$

**Test for Solutions on a Number Line:**



$$\begin{aligned} 5 - \sqrt{104} &\geq -3 \\ 5 - 10.1 &\geq -3 \\ -5.1 &\geq -3 \end{aligned}$$

$$-\frac{1}{5} \leq x \leq 3$$

Solve the following:

1.  $3\sqrt[3]{2x-1} + 6 < 15$

$\frac{1}{2} \leq x < 5$

$3\sqrt[3]{2x-1} < 9$

$\sqrt[3]{2x-1} < 3$

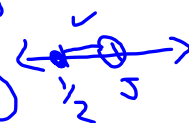
$2x-1 < 27$

$2x < 28$

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$



2.  $\sqrt{2x+8} - 4 > 2$

$2x+8 \geq 0$

$2x \geq -8$

$x \geq -4$

$\sqrt{2x+8} > 6$

$2x+8 > 36$

$2x > 28$

$x > 14$



4.  $4\sqrt{b+3} - \sqrt{b-2} \geq 10$

$b \geq 6$

3.  $5\sqrt[3]{x+2} - 8 < 2$

$x < 6$

Radicand may be negative.

$\sqrt[3]{x+2} < 2$

$x+2 < 8$

$x < 6$

5.  $\sqrt{2d+1} + \sqrt{d} \leq 5$

$0 \leq d \leq 4$

$2d+1 \geq 0$

$d \geq -\frac{1}{2}$

$(\sqrt{2d+1})^2 \leq (5-\sqrt{d})^2$

$2d+1 \leq 25-10\sqrt{d}+d$

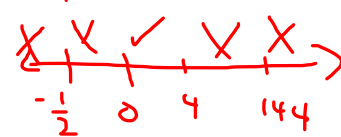
$(d-24) \leq (-10\sqrt{d})^2$

$d^2-48d+576 \leq 100d$

$d^2-148d+576 \leq 0$

$(d-144)(d-4) \leq 0$

test



$d=144$   $d=4$

Try These:

1.  $\sqrt{z+5} + 4 \leq 13$

2.  $8 + \sqrt{2q} \leq 5$