5.8 Solving Radical Inequalities

Radical Inequality: Inequality that has a variable in the radicand.

To Solve Radical Inequalities:

Step 1	If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.
Step 2	Solve the inequality algebraically.
Step 3	Test values to check your solution.

Why?

Example **Solve** $5 - \sqrt{20x + 4} \geq -3$

Since the radicand of a square root must be greater than or equal to zero, first solve

$20x + 4 \geq 0.$

$20x + 4 \geq 0$

$20x \geq -4$

$x \geq -\frac{1}{5}$

Now solve $5 - \sqrt{20x + 4} \geq -3.$

$5 - 5 \geq -3$

$5 - 10.1 \geq -3$

$-5.1 \geq -3$

$5 - \sqrt{20x + 4} \geq -3$	Original inequality
$\sqrt{20x + 4} \leq 8$	Isolate the radical.
$20x + 4 \leq 64$	Eliminate the radical by squaring each side.
$20x \leq 60$	Subtract 4 from each side.
$x \leq 3$	Divide each side by 20.

$-\frac{1}{5} \leq x \leq 3$

Test for Solutions on a Number Line:

Solve the following:

1. $3\sqrt{2x-1}+6<15$

$\frac{1}{2} \le x < 5$

$3\sqrt{2x-1} < 9$

$\sqrt{2x-1} < 3$

$2x-1 < 9$

$x < 5$

$2x-1 \ge 0$

$2x \ge 1$

$x \ge \frac{1}{2}$

2. $\sqrt{2x+8}-4>2$

$2x+8 \ge 0$

$2x \ge -8$

$x \ge -4$

$\sqrt{2x+8} > 6$

$2x+8 > 36$

$x > 14$

$x \ge 14$

3. $5\sqrt[3]{x+2}-8<2$

$x < 6$

Radicand may be negative.

$\sqrt[3]{x+2} < 2$

$x+2 < 8$, $x < 6$

4. $4\sqrt{b+3}-\sqrt{b-2} \ge 10$

$b \ge 6$

5. $\sqrt{2d+1}+\sqrt{d} \le 5$

$0 \le d \le 4$

$2d+1 \ge 0$, $d \ge -\frac{1}{2}$

$d \ge 0$

$(\sqrt{2d+1})^2 \le (5-\sqrt{d})^2$

$2d+1 \le 25-10\sqrt{d}+d$

$(d-24)^2 \le (-10\sqrt{d})^2$

$d^2-48d+576 \le 100d$

$d^2-148d+576 \le 0$

$(d-144)(d-4) \le 0$

$d = 144$ $d = 4$

test

Try These:

1. $\sqrt{z+5}+4 \leq 13$

2. $8+\sqrt{2q} \leq 5$